The Coin Lab

Introduction

What determines the numbers that are selected in the lottery or whether heads or tails results after a coin flip? We can ask similar questions about biological events. What determines whether chromosome 14 will be pulled to the left side of the cell or the right side during anaphase I of meiosis? The answer is chance.

Despite common understandings of chance as something completely unpredictable, in math and science chance is expressed through probability in precise mathematical terms that place predictable limits on chance events. Specifically, probability is defined as the likelihood of a particular chance event occurring among the total number of equally likely chance events.

For a coin toss, we can calculate the probability that heads will result from one toss. If heads is the number of particular chance events of interest, then the numerator is simply “1.” The total number of equally likely events is “2” because tails is just as likely as heads. Thus, the probability is ½ or 50 percent.

We can now ask, what is the probability of obtaining heads if we flip the coin a second time? The *First Law of Probability* states that the results of one chance event have no effect on the results of subsequent chance events. Thus, the probability of obtaining heads the second time you flip it remains at ½. Even if you obtained five heads in a row, the odds of heads resulting from a sixth flip remain at ½.

What if we ask a different question, what is the probability of flipping a coin twice and obtaining two heads (or, equivalently, flipping two coins at the same time)? Notice that this is different from the previous question; no coins have been flipped yet. In this case, we turn to the *Second Law of Probability* (or the Rule of Multiplication), which states that the probability of independent chance events occurring together is the *product* of the probabilities of the separate events. Thus, if the probability of one coin coming up heads is ½, and the independent likelihood of the second coin coming up heads is ½, then the likelihood that both will come up heads is ½ x ½ = ¼.

The *Third Law of Probability* (the Rule of Addition) considers what happens with chance events that are mutually exclusive, which means that they are not independent. In other words, if one event occurs then another event cannot occur. This frequently applies to combinations. For example, we could ask, What is the probability that in three coin flips, two heads and one tail will result? There are three possibilities for how this might occur:

Possibility 1: head-head-tail

Possibility 2: head-tail-head

Possibility 3: tail-head-head

If sequence 2 comes up, neither 1 nor 3 can happen, although in advance you have no way of knowing which might happen. All three sequences are equally likely, however. To calculate this, use the *Third Law* which states that the probability that any of a number of mutually exclusive events will occur is the *sum* of their independent probabilities. For the example above:

P(1) = ½ x ½ x ½ = 1/8

P(2) = ½ x ½ x ½ = 1/8

P(3) = ½ x ½ x ½ = 1/8

P(total) = 3/8

Thus, the probability of obtaining two heads and one tail in three separate coin flips is 3/8. These same rules of probability allow us to calculate the odds of parents conceiving particular numbers of girls or boys or of predicting the likelihood that specific chromosomes will segregate together into the same gamete. All are chance events.

STOP: Please answer the four questions in the Pre-Lab Quiz

Binomial Expansion:

Assume that a couple plans to have five children. In this case, it is somewhat tedious to outline and then calculate all the possible combinations because the number of independent events has increased beyond three or four.

In order to predict the probability of any event, especially when large numbers of events are concerned, it is convenient to use an expansion of the binomial theorem: $(a + b)^n$, where "a" = the chance of a certain event happening (heads), "b" = the chance of the alternative event happening (tails), and "n" = the number of individuals concerned in the event.

Example: If 2 coins are tossed together twice in succession, the total number of individuals concerned (n) with each event is 2, and the binomial expanded:

$$(a + b)^2 = a^2 + 2ab + b^2 = 1$$

Thus, a^2 = the chance of both coins showing heads = ½ x ½ = ¼
b^2 = the chance of both coins showing tails = ½ x ½ = ¼ ; and
$2ab$ = the chance of one turning heads and the other tails is 2 x ½ x ½ = ½

In the same way, if 3 coins are tossed simultaneously, the binomial is expanded as:

$$(a + b)^n = (a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 = 1$$

where $3a^2b$ and $3\ ab^2$ are the frequencies for 2 heads and 1 tail and 1 head and 2 tails respectively.

If 4 coins are tossed simultaneously, the binomial expansion is:

$$(a + b)^n = (a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4 = 1$$

Example: The chance that out of 4 tossed coins, 3 will land heads and 1 will land tails is $4a^3b = 4 \times (1/2)^3 \times$ ½ = ¼ . This means that if 4 coins are tossed together 32 times, the combination of 3 heads and 1 tails will be expected to occur in ¼ of the 32 tosses, or in 8 tosses.

STOP: Please answer the next two Pre-lab Questions on Binomial Expansions

Goodness of Fit/Chi Square Test

Nevertheless, in actual practice, the observed frequencies of any event may deviate from the frequencies predicted by theory and it is necessary to determine whether the deviation is due to chance or some other factor (in this case, a two-headed coin, imperfectly balanced coin, etc.). The Chi-Square(X^2) test is used to determine how well the observed frequencies of an event fit with the theoretical calculated (expected) frequencies. X^2 is a constant which indicates the degree of deviation. From the value of X^2, it is possible to determine the percentage of cases in which such a deviation may be expected by chance.

$$X^2 = \sum \frac{(\text{Observed frequency} - \text{Expected frequency})^2}{\text{Expected frequency}}$$

o = observed frequency of event
e = calculated frequency of event (expected)
∑ = sum of the calculated valued for all classes or events.

The value for X^2 is then compared to values in a standard Chi-Square Table which gives the expected frequency (P) of the observed deviation as chance occurrence.

Example: 2 coins tossed together 12 times

Classes or events	Observed frequency	Expected	Deviation (o-e)	$\frac{(o-e)^2}{e}$	P value
2 heads	2	$a^2 \times 12 = 3$	-1	.333	
2 tails	3	$b^2 \times 12 = 3$	0	.000	
1 head/1 tail	7	$2ab \times 12 = 6$	+1	.166	
Totals	12	12		$X^2 = .499$	

X^2 with a value of .499 with 2 degrees of freedom (number of levels or independent variable-- in this case coin toss outcomes minus 1) corresponds to a P value of between 70-80%. This means that the deviation from the expected ratio will occur in more than 70% of the trials by chance alone. Therefore, there is no reason to believe that the deviation was due to anything more than chance factors. If the P value had been below 5%, it would be considered significantly low to indicate that the deviation had not been the result of chance. If the P value had been below 1%, it would be a highly significant indication that the deviation had not occurred by chance.

You will be using this probability test several times through the course. Some of the experiments performed will have to be tested mathematically. It is also extensively used in genetics in the calculations of the expected frequency of a given type of offspring. Therefore, it is based on the postulation of a particular mechanism for the inheritance of a trait and can be used to modify or abandon your inheritance hypothesis. The Chi Square test is frequently used to determine the significance of these deviations.

You are to perform the three experiments listed below.

a. Toss one coin 20 times
b. Toss two coins together 20 times
c. Toss three coins together 20 times

For each experiment:

1. Calculate the expected values for each possible outcome
2. Record your hypothesis for each set of coin tosses (a-c) – question 7 on attached question sheet
3. Flip coins and record your observed data on the attached Coin Probability and Chi-Squared Lab -Individual Group Data Sheet (the first one in this packet).
4. Calculate the observed - expected values and then determine the X^2 values (refer to previous section in this lab if you don't recall how to do this).
5. Determine the P values from the table provided (determine the degrees of freedom—for example, on the one coin toss this would be 1, two coin tosses would be 2 and three coin tosses would be 3).
6. Record your observed frequencies (o) on the "master sheet" of class data. From these, a X^2 will be calculated for the entire class (we'll record this on the Class data sheet next class). You will need this information to answer question 8 on the analysis questions.

Chi Square Probability Values									**Reject**
DF	**P= .99**	**.95**	**.90**	**.75**	**.50**	**.25**	**.10**	**.05**	**.01**
1	0.00016	0.00393	0.01579	0.10153	0.45494	1.32330	2.70554	3.84146	6.63490
2	0.02010	0.10259	0.21072	0.57536	1.38629	2.77259	4.60517	5.99146	9.21034
3	0.11483	0.35185	0.58437	1.21253	2.36597	4.10834	6.25139	7.81473	11.3448
4	0.29711	0.71072	1.06362	1.92256	3.35669	5.38527	7.77944	9.48773	13.2767
5	0.55430	1.14548	1.61031	2.67460	4.35146	6.62568	9.23636	11.0705	15.0862

Coin Probability and Chi-Squared Lab Data
Individual Group Data

a. One coin tossed 20 times

Outcomes	(o)	(e)	(o-e)	$\frac{(o-e)^2}{e}$	X^2	P value (from table)
1 head					sum of column to the left	
1 tail						
Total						

Does the P-value allow you to Accept or Reject the data?____________________________

b. Two coins tossed 20 times

Outcomes	(o)	(e)	(o-e)	$\frac{(o-e)^2}{e}$	X^2	P value (from table)
2 heads						
2 tails					sum of column to the left	
1 head/1 tail						
Total						

Does the P-value allow you to Accept or Reject the data?____________________________

c. Three coins tossed 20 times

Outcomes	(o)	(e)	(o-e)	$\frac{(o-e)^2}{e}$	X^2	P value (from table)
3 head						
3 tail						
1 head/2 tails					sum of column to the left	
2 heads/1 tail						
Total						

Does the P-value allow you to Accept or Reject the data?____________________________

Coin Probability and Chi-Squared Lab Data
Class Data

a. One coin tossed 20 times

Outcomes	(o)	(e)	(o-e)	$\frac{(o-e)^2}{e}$	X^2	P value (from table)
1 head					sum of column to the left	
1 tail						
Total						

Does the P-value allow you to Accept or Reject the data?___________________________

b. Two coins tossed 20 times

Outcomes	(o)	(e)	(o-e)	$\frac{(o-e)^2}{e}$	X^2	P value (from table)
2 heads						
2 tails					sum of column to the left	
1 head/1 tail						
Total						

Does the P-value allow you to Accept or Reject the data?___________________________

c. Three coins tossed 20 times

Outcomes	(o)	(e)	(o-e)	$\frac{(o-e)^2}{e}$	X^2	P value (from table)
3 head						
3 tail						
1 head/2 tails					sum of column to the left	
2 heads/1 tail						
Total						

Does the P-value allow you to Accept or Reject the data?___________________________

Coin Lab Questions

Pre-Lab Questions 1-4 will be given as a short quiz before beginning this lab. These questions will relate to the three Laws of Probability described in the Introduction section.

A few more Pre-lab practice problems- Using Binomial Expansions

5. A couple would like to have three children. They are hoping to have two boys and a girl. What is the probability that they will get this combination of children?

6. Now let's examine a couple that is planning a large family. They plan to have three children, and they would like to know the probability that all three will be boys; all three girls; two boys/one girl; one boy/two girls. Now predict the expected numbers of each class if 100 couples have three children each.

Hypothesis

7. Write the hypotheses that you were testing (perhaps unknowingly) in the experiments that produced the data in tables A-C. Also, assuming each lab pair in the class carries out the experiment, make a hypothesis for class data in tables D-F.

Coin Lab Analysis (Post Lab) Questions:

8. How close were the class ratios for each trial to the ratios predicted by the binomial theorem? *Be specific. You'll need tabulated class data to answer this question.*

9. Interpret your Chi Square values (X^2) and comment on whether you should support or reject your hypotheses?

10. Why was the Chi Square test used in the coin lab? What does it tell you about your data? *Be specific, research this answer if necessary*

11. What does the data (both your own and the classes) say about designing experiments in terms of sample size and the accuracy of calculating ratios? *Explain.*

12. Relate the results from your coin lab to real world genetics crosses. Explain how flipping the coin models meiosis and use a hypothetical example of show your understanding of probability and genetics.